$\textbf{Q-R-pre-preserving_functionality_wrt_implies}^{11,40}$

```
\begin{split} \forall es\text{:ES, } P_1, \, P_2\text{:}(\to \mathbb{P}), \, Q_1, \, R_1, \, Q_2, \, R_2\text{:}(\to \to \to \mathbb{P}), \, f\text{:}(\{e\text{:E}|\ P_1(e)\} \to \to). \\ P_1 &\leftarrow P_2 \\ &\Rightarrow Q_1 \leftarrow Q_2 \\ &\Rightarrow R_1 => R_2 \\ &\Rightarrow \{f \text{ is } Q_1\text{-}R_1\text{-pre-preserving on } P_1 \Rightarrow f \text{ is } Q_2\text{-}R_2\text{-pre-preserving on } P_2\} \end{split}
```